Chiral Extrapolation of the Strangeness Changing Scalar $K\pi$ Form Factor

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Abstract: We perform a chiral extrapolation of lattice data on the scalar $K\pi$ form factor and the ratio of the kaon and pion decay constants within Chiral Perturbation Theory to two loops. We determine the value of the scalar form factor at zero momentum transfer, at the Callan-Treiman point and at its soft kaon analog as well as its slope. Results are in good agreement with their determination from experiment using the standard couplings of quarks to the W boson. The slope is however rather large. A study of the convergence of the chiral expansion is also performed.

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1 Introduction

In recent years lots of progress has been made in QCD lattice calculations. One important progress in the light quark sector concerns the values of the quark masses that can now be reached. These are very close to the physical ones making a controlled, i.e. trustable, chiral extrapolation of the lattice results to the physical points possible. A very powerful model-independent framework to perform this extrapolation is Chiral Perturbation Theory (ChPT), the Effective Field Theory of QCD at low energies. Indeed it allows to calculate low-energy QCD processes in terms of the light pseudoscalar mesons masses. Hadron properties are presently actively studied on the lattice and chiral extrapolations to their physical values are performed, see for example [1, 2, 3].

At the same time lots of effort is put into testing the Standard Model (SM). In order to do so one has to have very precise determinations of the QCD quantities which generally enter the different processes under consideration. Two very interesting quantities in this respect are the strangeness changing scalar f_0 and vector f_+ form factors which are measured in K_{l3} decays [4]. Indeed a measurement of the $K_{\ell 3}$ inclusive decay rate leads to the extraction of the product of the vector form factor at zero momentum transfer $f_+(0)$ and of the CKM matrix element $|V_{us}|$. Consequently the knowledge of $f_+(0)$ allows to extract this matrix element and thus to test the unitarity relation between the elements of the first row of this matrix. Another test comes from the values of this form factor at the Callan-Treiman point [5] and at its soft-kaon analog [6]. Indeed, at these particular points the scalar form factor has a well-known value as dictated by $SU(N_f) \times SU(N_f)$ low energy theorems, with $N_f = 2$ and $N_f = 3$, respectively. Combining this information with experimental results from semi-leptonic decays one can determine the values of the scalar form factor at these two points in the SM. Thus a departure from these values would be a sign for physics beyond the SM such as right-handed quark couplings to the W [5, 7] or charged Higgs effects, see for example the discussion in [4](and references therein) and [8]. However, in order to have a reliable and accurate test of the SM one should know very precisely the corrections to the Callan-Treiman theorem and its soft kaon analog which are only exact in the soft meson limit. They are usually calculated in ChPT [9]. In Ref. [6] the one-loop result from Ref. [9] was used and an estimation of the higher order effects was done since at next-to-leading order some low-energy constants (LECs) contribute which are not very precisely known at present. Experimentally there has recently been interest in trying to obtain the value of the scalar form factor at the Callan-Treiman point. The three collaborations NA48 [10], KLOE [11] and KTeV [12] have reanalysed their data so as to extract this value using in their analysis a dispersive representation of the form factors proposed in Refs. [5, 6]. With the current experimental precision the data from the last two collaborations show a good/marginal agreement with the SM while NA48 has a 4.5σ deviation.

The scalar form factor has been studied on the lattice. Some parameterization of its momentum-dependence plus the knowledge of the one-loop ChPT result at zero momentum transfer is used to extract $f_+(0)$. Here we will fit the lattice data from Ref. [13] for the scalar form factor using a ChPT calculation at two loop order [14]. Furthermore, we will not only consider the scalar form factor but at the same time we will fit the ratio of the kaon to the pion decay constants F_K/F_π [15, 16] since, as we will see, similar LECs enter the two quantities. This will allow us to determine some

LECs at two-loop order ($\mathcal{O}(p^6)$) and thus not only obtain $f_+(0)$ and determine $|V_{us}|$ but also the value of the scalar form factor at the Callan-Treiman point and at its soft-kaon analog. Of course one should keep in mind that we are dealing here with $SU(3)$ quantities which involve the strange quark mass. The question is whether one should consider the strange quark as light compared to the QCD scale $\Lambda \sim 200$ MeV or should it be treated as heavy. Related to that is the question whether standard $SU(3)$ ChPT which assumes that the quark condensate is large, is a well converging series, the relevant expansion parameter being in that case $(m_K/\Lambda_\chi)^2 \sim 0.4^2$. Also $\bar{s}s$ sea quark pairs may play a significant role in chiral dynamics leading to different patterns of chiral symmetry breaking in $N_f = 2$ and $N_f = 3$ chiral limits [17, 18]. For example, lattice QCD seems to indicate a problem in the extrapolation of F_K/F_π to its physical value when using $SU(3)$ ChPT to one loop order [16] while a fit within "Kaon ChPT" [19] where the kaon is treated as a heavy particle leads to good agreement. The mass dependence of the scalar form factor has been studied within this scheme in Ref. [20]. We will use here standard ChPT to two loops and we will study the convergence of the chiral expansion. We will also discuss the leading order $\mathcal{O}(p^4)$ LEC L_4^r which is related to the Okubo-Zweig-Iizuka (OZI) rule violation.

In section 2, we discuss briefly the scalar form factor at two loops in ChPT. We present the lattice calculations in section 3 and discuss our fits and results in section 4. We conclude in section 5.

2 ChPT to two loops

The strangeness changing form factors are defined from the $K \to \pi$ matrix element of the vector current $V_\mu = \bar{s}\gamma_\mu u$

$$\langle \pi(p_\pi)|\bar{s}\gamma_\mu u|K(p_K)\rangle = (p_\pi + p_K)_\mu \, f_+(t) + (p_K - p_\pi)_\mu \, f_-(t), \tag{2.1}$$

where $t \equiv q^2 = (p_K - p_\pi)^2$. The vector form factor $f_+(t)$ represents the P-wave projection of the crossed channel matrix element $\langle 0|\bar{s}\gamma_\mu u|K\pi\rangle$ whereas the S-wave projection is described by the scalar form factor defined as

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t) \; . \tag{2.2}$$

At zero momentum one has

$$f_0(0) = f_+(0) \; . \tag{2.3}$$

These form factors were calculated to two loops in ChPT in Ref. [14]. These authors introduced the quantity

$$\tilde{f}_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2}\left(f_-(t) + 1 - \frac{F_K}{F_\pi}\right) = f_0(t) + \frac{t}{m_K^2 - m_\pi^2}\left(1 - \frac{F_K}{F_\pi}\right) . \tag{2.4}$$

The two-loop expressions of the two decay constants F_K and F_π can be found in Ref. [15]. They involve two L_i, L_4^r and L_5^r, at $\mathcal{O}(p^4)$ and the four $\mathcal{O}(p^6)$ C_i, C_{14}^r, C_{15}^r, C_{16}^r and C_{17}^r. Assuming that the

LEC L_4^r is small, which is in principle the case in the standard scenario of ChPT, one can expand, as usually done, the denominator in the ratio of the two decay constants so that its contribution to order p^4 cancels and one is left with the contribution from L_5^r and two combinations of three C_i as detailed below:

$$\begin{aligned} F_K/F_\pi &= 1+\frac{4}{F_\pi^2}(m_K^2-m_\pi^2)L_5^r+\frac{8}{F_0^2}\Big[-m_\pi^4(C_{15}^r+2C_{17}^r) \\ &+ 2m_\pi^2 m_K^2\left(-(C_{14}^r+C_{15}^r)+\frac{1}{2}(C_{15}^r+2C_{17}^r)\right)+2m_K^4(C_{14}+C_{15})\Big]+\delta\ , \end{aligned} \tag{2.5}$$

where F_0 is the pion decay constant in the chiral limit. We will come back to the discussion of this equation in Section 4. δ contains the loops and the contributions of the L_i at $\mathcal{O}(p^6)$. Interestingly the dependence on these LECs is exactly the same in $f_0(t)$, see Ref. [14]. Thus the main advantage in considering $\tilde{f}_0$ is that this quantity has no dependence on the L_i^r at order p^4, only via order p^6 contributions and furthermore, it only depends on the two $\mathcal{O}(p^6)$ LECs C_{12}^r and C_{34}^r. Its explicit dependence on those is given by

$$\begin{aligned} \tilde{f}_0(t) &= 1-\frac{8}{F_0^2}\left(C_{12}^r+C_{34}^r\right)\left(m_K^2-m_\pi^2\right)^2+8\frac{t}{F_0^2}\left(2C_{12}^r+C_{34}^r\right)\left(m_K^2+m_\pi^2\right) \\ &-\frac{8}{F_0^2}t^2C_{12}^r+\overline{\Delta}(t)+\Delta(0)\,, \end{aligned} \tag{2.6}$$

where we used the notations of Ref. [14]. As before, the quantities $\overline{\Delta}(t)$ and $\Delta(0)$ have contributions from loops and from the LECs L_i^r at $\mathcal{O}(p^6)$ and can in principle be calculated to order p^6 accuracy with the knowledge of the L_i^r to order p^4. Parameterizations of these quantities in the physical region of $K_{\ell 3}$ decays can be found in Ref. [14].

Eq. (2.4) is in fact inspired by the Callan-Treiman theorem [21] which predicts the value of $f_0(t)$ at the so called Callan-Treiman point, $t\equiv\Delta_{K\pi}=m_K^2-m_\pi^2$ in the $SU(2)\times SU(2)$ chiral limit. One has

$$f_0(\Delta_{K\pi})=\frac{F_K}{F_\pi}+\Delta_{CT}\ , \tag{2.7}$$

where Δ_{CT} is a correction of $\mathcal{O}\left(m_{u,d}\right)$. It has been estimated within ChPT at next-to-leading order (NLO) in the isospin limit [9] with the result

$$\Delta_{CT}^{NLO}=(-3.5\pm 8.0)\cdot 10^{-3}\ , \tag{2.8}$$

where the error is a conservative estimate assuming some typical corrections of $\mathcal{O}(m_{u,d})$ and $\mathcal{O}(m_s)$. From Eq. (2.6) one can calculate the contribution from the $\mathcal{O}(p^6)$ LECs to Δ_{CT}. It reads

$$\Delta_{CT}|_{C_i}=\frac{16}{F_0^2}(2C_{12}^r+C_{34}^r)m_\pi^2(m_K^2-m_\pi^2)\,. \tag{2.9}$$

$\mathcal{O}(p^6)$ calculations [22] using some estimates for the LECs C_{12}^r and C_{34}^r give results consistent with Eq. (2.8). Strong isospin breaking as well as electromagnetic effects have also been evaluated [22, 23].

Another interesting quantity is the soft-kaon analog of the Callan-Treiman theorem [24]

$$f_0(\tilde{\Delta}_{K\pi}) = \frac{F_\pi}{F_K} + \tilde{\Delta}_{CT} \,, \tag{2.10}$$

with $\tilde{\Delta}_{K\pi} \equiv -\Delta_{K\pi}$. A one loop calculation of the $SU(3)$ correction $\tilde{\Delta}_{CT}$ in the isospin limit [9] gives $\tilde{\Delta}_{CT} = 0.03$. This is larger than its soft-pion analog Δ_{CT}, see Eq. (2.8), by a factor m_K^2/m_π^2, however, rather small for a first order $SU(3) \times SU(3)$ breaking effect, which is expected to be of the order of about 25%.

The value of V_{ud}, the first element of the CKM matrix is very accurately known from superallowed $0^+ \to 0^+$ nuclear β-decays [25]

$$|V_{ud}| = 0.97425 \pm 0.00022 \,. \tag{2.11}$$

Combining this value with the experimental value of the branching ratio $\Gamma_{K_{l2(\gamma)}}/\Gamma_{\pi_{l2(\gamma)}}$ [26] and assuming the standard couplings of quarks to the W-boson allows to determine the ratio of the decay constants F_K/F_π. Using instead the inclusive decay rate $\Gamma_{K_{Le3(\gamma)}}$ [26], one obtains the value of the vector form factor at zero momentum transfer $f_+(0)$. From these information and Eqs. (2.7), (2.10), one can deduce the value of the normalized form factor at the Callan Treiman point $C \equiv f_0(\Delta_{K\pi})/f_+(0)$ and at $\tilde{\Delta}_{K\pi}$. For the explicit formulae and more details see for example Ref. [26]. One has the following updated values in the SM

$$\begin{aligned}
&f_+(0)|_{SM} = 0.959 \pm 0.005 \,, \\
&F_K/F_\pi|_{SM} = 1.192 \pm 0.006 \,, \\
&\ln C|_{SM} = 0.2169 \pm 0.0034 + \Delta_{CT}/f_+(0) \,, \\
&f_0(\tilde{\Delta}_{K\pi})/f_+(0)|_{SM} = 0.8302 \pm 0.0074 + \tilde{\Delta}_{CT}/f_+(0) \,.
\end{aligned} \tag{2.12}$$

Deviations from these SM predictions would thus be a sign of new physics. For example at NLO within the minimal not-quite decoupling electroweak low-energy effective theory (LEET) [27], in the light quark sector one has two combinations of parameters of spurionic origin describing the couplings of quarks to the W-boson to be determined from experiment [28, 6]. While the knowledge of the scalar form factor at the CT point measures one combination, its knowledge at $\tilde{\Delta}_{K\pi}$ measures the other one. A precise determination of Δ_{CT} and $\tilde{\Delta}_{CT}$ would thus help to settle the issue of the presence of right-handed couplings of quarks to the W-boson.

In order to have a very precise determination of $f_+(0)$ as well as Δ_{CT} and $\tilde{\Delta}_{CT}$, one needs to have a very precise determination of all the LECs L_i^r and C_i^r which enter Eqs. (2.5, 2.6).

- The L_i have been determined in Ref. [29] from a fit to the masses and to K_{l4}-decay data from the E865 experiment, assuming that L_4^r and L_6^r are $1/N_c$ suppressed and using $F_K/F_\pi = 1.22$ [3]. Matching the dispersive results for the subthreshold expansion parameters of πK scattering

[3] In this fit some of the C_i are taken from resonance saturation, the others are set to zero, see Ref. [15]

	Fit 10 [29] set a	πK Roy Steiner [30]	Prelim. Fit All(*) [45] set b	Lattice [16]
$10^3 L_1^r$	0.432	1.05 ± 0.12	0.99 ± 0.13	–
$10^3 L_2^r$	0.735	1.32 ± 0.03	0.60 ± 0.21	–
$10^3 L_3^r$	-2.35	-4.53 ± 0.14	-3.08 ± 0.47	–
$10^3 L_4^r$	0	0.53 ± 0.39	0.70 ± 0.66	$0.33(0.13)$
$10^3 L_5^r$	0.97	3.19 ± 2.40	0.56 ± 0.11	$0.93(0.073)$
$10^3 L_6^r$	0		0.14 ± 0.70	-
$10^3 L_7^r$	-0.31		-0.21 ± 0.15	-
$10^3 L_8^r$	0.6		0.38 ± 0.17	-
$10^3(2L_6^r - L_4^r)$				0.032 (0.062)
$10^3(2L_8^r - L_5^r)$				0.050(0.043)

Table 1: $\mathcal{O}(p^4)$ LECs at a scale $\mu = 0.77$ GeV.

with their chiral expansion at order p^4 [30] leads to somewhat different results, especially L_4^r is suggestive of a significant violation of the OZI rule in the scalar sector, see Table 1. This is in agreement with a determination of some of the LECs in an analysis of J/ψ decays into vector mesons and two pseudoscalars [31].

- In Ref. [32] it was shown that it was possible to reproduce the values of the L_i in terms of properties of the light meson resonances (masses and coupling constants). The idea of using resonance saturation also for the $\mathcal{O}(p^6)$ LECs was thus taken up and the C_i are presently mostly estimated in that framework [33, 34, 35]. There are, however, a few problems. First the scale at which they are obtained is not known. It is usually assumed to be given by the lightest scalar nonet that survives in the large N_c limit, $M_S = 1.48$ GeV. The value at another scale, typically the ρ mass scale, is obtained using renormalization group equations. Furthermore, a test of the naturalness of the C_i [36] shows that some of them are in fact not dominated by the resonance contributions. Also the LECs we are interested in have important contributions from the scalar sector where one knows that the OZI rule is strongly violated and where the presence of the wide scalar σ and κ mesons makes the calculation in terms of tree level diagrams from a resonance Lagrangian not really appropriate. Considering more specifically C_{12}^r and C_{34}^r, several calculations have been performed based on the study of the scalar form factor with $\Delta S = 0$ [37] or $\Delta S = 1$ [38, 39]. In the literature these two LECs, Eq. (2.6), lie in the range -10^{-3} GeV^{-2} to a few 10^{-4} GeV^{-2}. The four other $\mathcal{O}(p^6)$ LECs (C_{14}^r, C_{15}^r C_{16}^r and C_{17}^r), Eq. (2.5), needed in our study are not very well known. In Ref. [40] where the C_i have been recently determined within a quark model, one finds $C_{15}^r = C_{16}^r = 0$, $C_{17}^r = 0.01 \cdot 10^{-3}$ GeV^{-2} and $C_{14}^r = -0.83 \cdot 10^{-3}$ GeV^{-2} which is smaller than what is found in resonance saturation $C_{14}^r = -4.3 \cdot 10^{-3}$ GeV^{-2}.

With the progress of lattice QCD it becomes also possible to extract the LECs from a chiral extrapolation of the lattice data. Already some of the $\mathcal{O}(p^4)$ ones have been obtained mostly within $SU(2)$ (l_i). Relations between the $SU(2)$ and the $SU(3)$ LECs allows to determine the L_i from the l_i [41, 42]

(for similar relations between the C_i see Ref.[43]). Results from the RBC/UKQCD collaboration are shown in Table 1. As can be seen from this table most of the $\mathcal{O}(p^4)$ LECs are still not well enough determined for a very precise test of the SM. A global fit of all the low-energy constants of Chiral Perturbation theory at next-to-next-to-leading order currently performed [44, 45] will hopefully help to settle the values of these LECs much more precisely. Some preliminary results [45] which differ from fit 10 by using some more recent data, by letting L_4^r and L_6^r free and by adding some constraints from πK scattering show better agreement with the analysis of Ref. [30] as the comparison between the second and third column of Table 1 shows.

3 Lattice

Following the pioneering work of Ref. [46] different collaborations have extracted the vector form factor at zero momentum transfer either with $N_f = 2$ [47, 48, 49, 50] or $N_f = 2 + 1$ [13] flavours. The idea is to first evaluate the scalar form factor $f_0(t)$ at the momentum transfer $t_{\max} = (m_K - m_\pi)^2$. This can be very efficiently done calculating a double ratio of three-point correlation functions [46]. Then a phenomenologically motivated interpolation is performed up to zero momentum transfer [4] and the Ademollo-Gatto theorem is used to obtain a rather precise value for $f_+(0)$. Let us consider the chiral expansion of $f_+(0)$

$$f_+(0) = 1 + f_2 + f_4 + \cdots \; , \tag{3.1}$$

where $f_n = \mathcal{O}((m_{K,\pi}/(4\pi F_\pi))^n)$ and the first term is equal to one due to gauge invariance. The Ademollo-Gatto theorem [51] states that the deviation from unity of $f_+(0)$ is predicted to be second order in $SU(3)$ symmetry breaking, *i.e.* of order $(m_s - \hat{m})^2$, where m_s and $\hat{m}$ are the strange and the average of the u, d quark masses, respectively [5] so that the $\mathcal{O}(p^2)$ term f_2 in the chiral expansion of $f_+(0)$ is free of any LECs. The different collaborations generally take this term from a one-loop ChPT calculation [9]

$$f_2 = -0.0227 \; , \tag{3.2}$$

obtained for pion, kaon and eta masses taken at their physical values and in the isospin limit and determine the difference

$$\Delta f = f_+(0) - 1 - f_2 \; . \tag{3.3}$$

This difference contains of course all terms starting at the order $\mathcal{O}(p^6)$. Also used is the partially quenched expression derived in Ref. [53]. An expression for f_2 using NLO SU(2) ChPT can be found in Ref. [20]. The first determination of Δf in a quark model framework gave $\Delta f = -0.016(8)$ [54].

The RBC/UKQCD collaboration for example [13, 16] simulates with $N_f = 2 + 1$ flavors of dynamical domain wall quarks. In order to determine $f_+(0)$, they performed a simultaneous fit to

[4] A new technique has been developed in Ref. [52] which will allow to directly simulate at $t = 0$ on the lattice.

[5] Note, however, that despite this theorem the light quark mass difference $m_u \neq m_d$ can modify $f_+(0)$ to first order.

both the t and quark mass dependences using the ansatz

$$f_0(t, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1(m_K^2 + m_\pi^2))}{1 - t/(M_0 + M_1(m_K^2 + m_\pi^2))^2} \,. \tag{3.4}$$

This formula motivated by the Ademollo-Gatto theorem has four fit parameters $A_0,\ A_1,\ M_0,\ M_1$ and f_2 is the NLO term, Eq. (3.3). They have also used a second order Taylor expansion as parameterization of the t-dependence of the form factor [6]. They obtain

$$f_+(0) = 0.9644 \pm 0.0033_{\rm stat} \pm 0.0037_{\rm syst} \,. \tag{3.5}$$

The same collaboration has also extracted the ratio F_K/F_π [16]

$$F_K/F_\pi = 1.205 \pm 0.018_{\rm stat} \pm 0.062_{\rm syst} \,. \tag{3.6}$$

A summary of other lattice results can be found in Refs. [26, 55]. In the unquenched simulations they fall in the range between 1.189 and 1.218 for the central value of F_K/F_π and between 0.956 and 0.968 for the one of $f_+(0)$. While the errors on the former are very small, they are larger on the latter. All these numbers should be compared to the Standard Model values, Eq. (2.12).

4 Chiral Extrapolation

We now turn to the central point of the paper, namely the chiral extrapolation of the lattice data on F_K/F_π and $f_+(0)$ based on the two-loop ChPT calculations [14]. We use the results from the RBC/UKQCD collaboration since this is the only collaboration which has calculated both these quantities with $N_f = 2+1$ flavors. We take the data performed on the $24^3 \times 64$ volume with an inverse lattice spacing of $a^{-1} = 1.73(3)$ GeV and a simulated strange quark mass, $am_s = 0.04$ close to its physical value. We do not correct for finite volume effects (FV) or lattice artefacts (LA). They have been estimated for F_K/F_π [16] where the error bars they quote for these effects are roughly equal (FV) or even larger (LA) than the statistical ones. We only included the statistical errors in our fits. Also we did not include the correlations between F_K and F_π since they are not available. Lattice results have been obtained for four values of the light quark masses which correspond to pion (first number in parenthesis) and kaon masses (second number) equal to $(0.329, 0.575)$ GeV (set (I)) $(0.416, 0.604)$ GeV (set (II)), $(0.556, 0.663)$ GeV (set (III)) and $(0.671, 0.719)$ GeV. Clearly, ChPT cannot be valid at too high pion and kaon masses so we completely discard the last set in our fits and mostly use sets (I) and (II). For each pion mass they have calculated the scalar form factor at five values of t going from ~ -0.4 GeV^2 to $t_{\rm max}$. Again for the fits we only use the three smallest absolute values of t.

[6] This parameterization and the pole one are usually assumed either in lattice calculations or in most of the experimental analyses. One should note, however, that the pole parameterization has no real physical motivation in the case of the scalar form factor. Also it has been shown [12, 6] that in order to get a very precise parameterization of the scalar form factor in the physical region of K_{l3} decay ($m_\ell^2 < t < t_{max}$), an expansion up to third order had to be done.

	Fit I	Fit II	Fit III	Fit IV	Fit V	Fit VI
C_{12}	5.77 ± 0.56	7.84 ± 0.58	4.69 ± 0.95	5.74 ± 0.95	4.69 ± 0.56	4.43 ± 0.88
C_{34}	2.54 ± 0.43	-1.28 ± 0.44	3.76 ± 0.95	1.07 ± 0.96	3.76 ± 0.43	3.50 ± 0.94
C_{14}	0^*	0^*	0.65 ± 1.38	0.71 ± 1.42	0.65^*	-0.93 ± 0.67
$2C_{17}$	0^*	0^*	0.31 ± 3.31	1.92 ± 3.36	0.31^*	4.16 ± 1.56
F_0	89.8 ± 0.1	69.2 ± 0.0	89.8 ± 0.1	69.3 ± 0.0	89.8^*	89.8 ± 0.1
$f_+(0)$	0.956	0.963	0.956	0.961	0.956	0.958
F_K/F_π	1.20	1.19	1.20	1.19	1.20	1.19
$\ln C$	0.22	0.20	0.22	0.21	0.22	0.21
$f_0(\tilde{\Delta}_{K\pi})$	0.75	0.75	0.75	0.76	0.75	0.77
$10^3\Delta_{CT}$	1.00	-2.14	0.27	-3.65	0.18	-0.32
$10^2\tilde{\Delta}_{CT}$	-9.00	-9.86	-8.24	-8.18	-8.11	-7.03
$10^3\lambda_0$	18.08	17.77	18.24	17.66	18.18	16.71
χ^2	1.40/4	0.96/4	1.67/4	1.29/4	3.01/4	4.8/7

Table 2: Result of the fits to the lattice data. The first five quantities are the parameters of the fits. The star denotes an input quantity. The C_i's are in units 10^{-4} GeV^{-2} and F_0 is in MeV. In Fits (I) and (II) the lattice data on F_K/F_π are not included.

A quantity $\mathcal{O}$ at two loops has typically the following form after renormalization of the pion decay constant

$$\mathcal{O}(m_\pi, m_K, m_\eta) = \mathcal{O}_{LO} + \frac{\mathcal{O}_{\rm NLO}}{F_\pi^2} + \frac{\mathcal{O}_{\rm NNLO}}{F_0^4} \ , \tag{4.1}$$

where $\mathcal{O}_{LO}$, $\mathcal{O}_{\rm NLO}$ and $\mathcal{O}_{\rm NNLO}$ are the contribution at leading order (LO), next-to-leading order (NLO) and next-to-next-to-leading order (NNLO), respectively. Here, F_π is the pion decay constant calculated at $\mathcal{O}(p^4)$ at the value of the pion mass and of the kaon mass under consideration and F_0 is the pion decay constant in the chiral $SU(3)$ limit. When working at the physical pion and kaon masses one usually replaces everywhere the decay constant by its physical value, since the difference is of higher order. This is the procedure which has been used to determine the LECs, set (a) and (b) of Table 1. If this is mostly justified for set (a) where the difference between F_π and F_0 is small, this is more questionable for set (b) where $F_0 = 67.1$ MeV but allows, of course, for a better convergence of the chiral series. Also going away from the physical point the difference between F_π and F_0 might become again too large for this procedure to be entirely satisfying. Here we will just replace F_0 by the physical value of F_π in the NNLO term in order to be consistent with the determination of the LECs. Also to be consistent with their determination we will use Eq. (2.5) for determining F_K/F_π. Again if this is justified for set (a) where the convergence of this quantity is rather good as we will see below, this is more questionable for set (b). In the expression Eq. (4.1), the mass of the η enters the NLO and NNLO terms. In the calculation of $\mathcal{O}_{\rm NLO}$ its NLO expression is used while in $\mathcal{O}_{\rm NNLO}$ the η mass is given by the Gell-Mann-Okubo relation.

We have performed several fits to the lattice data and determined from these fits results for $f_+(0)$, the slope of the scalar form-factor at zero momentum transfer λ_0, F_K/F_π, Δ_{CT} and $\tilde{\Delta}_{CT}$. We have taken F_0 as a parameter of the fit using the value of the physical pion decay constant as input. Apart

from Fit (VI) they are done with the two lattice data sets with the smallest pion values, sets (I) and (II). All the fits are done for the three smallest absolute values of t. The values of the LECs L_i are taken from sets (a) and (b) of Table 1. These sets correspond to a value of $m_s/\hat{m} = 24$. In Ref. [44] another preliminary set is given corresponding to a somewhat larger value $m_s/\hat{m} = 27.8$ as obtained by MILC and HPQCD/UKQCD. It leads to an even smaller value of $F_0 = 62.7$ MeV and will not be discussed here. The results of the fits are given in Table 2. The one from this other set are comparable to the one of set (b).

- Fits (I) and (II) are three parameter fits of $f_0(t)$ using sets (a) and (b) respectively. The C_i are the one used in the determination of the $\mathcal{O}(p^4)$ LECs, fit 10, namely $C_{14}^r = C_{15}^r = C_{16}^r = C_{17}^r = 0$. For set (a) $F_K/F_\pi = 1.22$ whereas for set (b) $F_K/F_\pi = 1.19$. Slightly different values are given in the table for set (a) since, as explained below Eq. (4.1) we did not use the physical value of F_π in the calculation of this quantity in the NLO term but rather its NLO expression.

- Fits (III) and (IV) are combined fits of F_K/F_π and $f_0(t)$ using sets (a) and (b), respectively, as in the previous fits but now the combinations $C_{14}^r + C_{15}^r$ and $C_{15}^r + 2C_{17}^r$ which appear in F_K/F_π are left free. Since we need to determine F_0, we, in principle, need to know C_{16}^r and the combination $C_{15}^r - 2C_{16}^r$. We will assume them equal to zero, this is consistent with the results in Ref. [40]. Thus we do in fact determine C_{14}^r and C_{17}^r.

- Fit (V): here we fix the combinations $C_{14}^r + C_{15}^r$ and $C_{15}^r + 2C_{17}^r$ from Fit (III) and we fit the quantity $\tilde{f}_0(t)$.

- Fit (VI) is the same as Fit (III) but with the lattice data for $f_0(t)$ from set (III) also included.

As can be seen from Table 2, we obtain very good fits of the lattice data. Fits (I) and (II), however, do not reproduce well the two lattice points for F_K/F_π from sets (I) and (II). Fits (III) and (IV) which correspond to two very different values of L_4^r are comparably good, but an order of magnitude larger value of C_{17}^r is in fact needed in order to compensate for the larger value of L_4^r in Fit (IV) compared to Fit (III). C_{14}^r and C_{17}^r are at least an order of magnitude smaller than what is expected from resonance saturation in the scalar sector which leads to typical values $\sim 10^{-3}$. One has for example [15]

$$C_{14} \sim \frac{c_d c_m d_m}{M_S^4} \sim -4.3 \cdot 10^{-3}\,\mathrm{GeV}^{-2} \tag{4.2}$$

where c_d, c_m and d_m are coefficients of the scalar chiral Lagrangian. M_S and d_m are obtained from the masses of the scalars $K_0^*(1430)$ and $a_0(980)$ and $c_m = 0.042$ GeV and $c_d = 0.032$ GeV. The results of Fit (VI) do not differ much from Fit (III), only C_{14} and C_{17} are larger in absolute value and the slope of the scalar form factor is somewhat smaller. This fit is shown on Fig. 1 for sets (I) and (III). Even though we only fit the three smallest points in absolute value, the t-dependence of set (III) is remarkably well reproduced by ChPT to two loops.

Fitting $f_0(t)$ leads to strong anticorrelations between C_{12} and C_{34} on the one hand and $C_{14} + C_{15}$ and $C_{15} + 2C_{17}$ on the other one, typically of the order of -0.8 while in Fit (V) the correlations

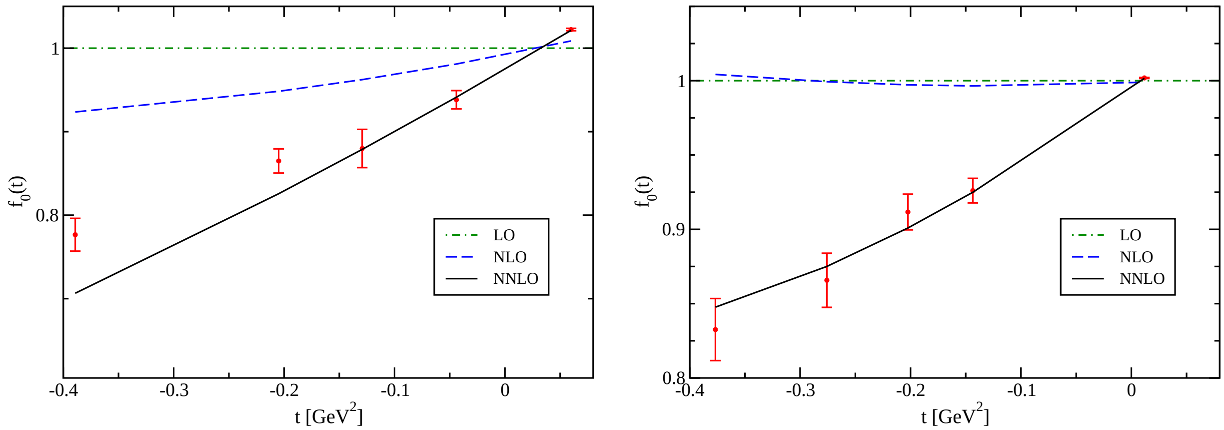

Figure 1: Momentum-dependence of the scalar form factor. The results of Fit (VI) are displayed (solid line) for set (I) (left panel) and set (III) (right panel). The convergence of the chiral expansion is also displayed: the dash-dotted line is the result at LO, the dashed line displays the one up to NLO.

between C_{12} and C_{34} are reduced by a factor of two. Also a comparison of Fits (III) and (V) shows that the error bars on these two LECs are smaller in the latter case. Thus a rather good determination of the LECs C_{12} and C_{34} is obtained by fitting the function $\tilde{f}_0$. Their order of magnitude is the one expected from resonance saturation. Note that the value obtained for $C_{12} + C_{34}$ is rather independent of the fits within one set, one gets $\sim 8 \cdot 10^{-4}$ GeV^{-2} for set (a) and $\sim 6 \cdot 10^{-4}$ GeV^{-2} for set (b).

The results for F_K/F_π, $f_+(0)$ and $\ln C$ are consistent with the values obtained assuming the standard quark couplings to the W-boson, Eq. (2.12). We refrain to give error bars here since one should have a more precise knowledge of the L_i as well as lattice data at lower pion and kaon masses to really be able to pin down these quantities very precisely. Difference between the various sets gives an idea of the errors. The value of λ_0 turns out to be rather large compared to the experimental results, the lattice determination of Ref. [50] or to what is obtained from the formula obtained in a dispersive parameterization of the form factor [5, 6]

$$\lambda_0 = \frac{m_\pi^2}{(m_K^2 - m_\pi^2)}(\ln C - G(0)), \qquad G(0) = 0.0398 \pm 0.0044\,, \tag{4.3}$$

where $G(t)$ is a dispersive integral of the phase of the form factor which is identified in the elastic region with the s-wave, $I = 1/2$ $K\pi$ scattering phase according to Watson's theorem. In the analysis [5, 6] it was taken from [30] where a matching of the solution of the Roy-Steiner equations with the $K\pi \to K\pi$, $\pi\pi \to K\bar{K}$ and $\pi\pi \to \pi\pi$ scattering data available at higher energies has been performed. Note that in this analysis the LECs obtained, second column of Table 1, are more consistent with the values used in Fit (IV), especially a large violation of the OZI rule was found. This large value of λ_0 can be traced back to the too large value of the combination $2C_{12} + C_{34}$ which enters its expression within ChPT, see Eq. (2.6). It is however compensated by a small curvature λ_0' leading to a value of the scalar form factor at the Callan-Treiman point in agreement with the SM value. Typically one obtains $\lambda_0' \sim 1 \cdot 10^{-4}$ instead of $\sim 6 \cdot 10^{-4}$ as expected from experiments and dispersive analyses

[56, 5]. Again C_{12} has a too large positive value. Stringent constraints on slope and curvature have recently been obtained using the method of unitarity bounds [57].

Let us study the convergence of the results. In Fig. 1 is shown $f_0(t)$ as obtained in Fit (VI) at LO (dot dashed line), NLO (dashed line) and NNLO (full line). On the left-hand-side (LHS) set (I) is displayed and on the right-hand-side (RHS) set (III), in order to compare the dependence on the pion and the kaon masses. Clearly, as expected, the convergence of $f_0(t)$ worsens as one increases the absolute value of t (LHS, set (I)) and as one increases m_π and m_K. At the physical pion and kaon masses one has from Fit (III),

$$\begin{aligned} f_+(0) &= 1 - 0.019 - 0.026 + \ldots, \\ F_K/F_\pi &= 1 + 0.140 + 0.061 + \ldots, \\ f_0(\Delta_{K\pi}) &= 1 + 0.139 + 0.063 + \ldots, \\ \Delta_{CT} &= 0 - 0.0025 + 0.0028 + \ldots, \\ \tilde{\Delta}_{CT} &= 0 + 0.024 - 0.106 + \ldots, \end{aligned} \tag{4.4}$$

and from Fit (IV)

$$\begin{aligned} f_+(0) &= 1 - 0.019 - 0.019 + \ldots, \\ F_K/F_\pi &= 1 + 0.113 + 0.081 + \ldots, \\ f_0(\Delta_{K\pi}) &= 1 + 0.110 + 0.081 + \ldots, \\ \Delta_{CT} &= 0 - 0.0033 - 0.0003 + \ldots, \\ \tilde{\Delta}_{CT} &= 0 + 0.021 - 0.103 + \ldots, \end{aligned} \tag{4.5}$$

where the first, second and third terms are the $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ contributions, respectively, and the ellipses denote terms of order p^8 and higher. Note that by definition Δ_{CT} and $\tilde{\Delta}_{CT}$ have no LO contribution. The convergence is rather good/not very good for F_K/F_π and $f_0(\Delta_{K\pi})$ for set (a) and set (b) respectively while the one for Δ_{CT} is good for set (b) and not for set (a). One should however keep in mind that the NLO correction for this last quantity is small being an $SU(2)\times SU(2)$ one. Also the NNLO contribution of Δ_{CT} is of the expected size of the corrections, Eq. (2.8). The convergence of $f_+(0)$ and $\tilde{\Delta}_{CT}$ is bad whatever the set. However the convergence looks again worth than it is in reality. Indeed for both quantities the contribution at NLO is smaller than naively expected. For $f_+(0)$ this is essentially due to the Ademollo-Gatto theorem, as we have seen in the previous section. In both cases the NNLO term is of the expected size. Let us look in a bit more details at the diverse contributions for $f_+(0)$, F_K/F_π and $f_0(\Delta_{K\pi})$. One has for Fit (III)

$$\begin{aligned} f_+(0) &= 1 + (-0.019 + 0.000) + (0.012 - 0.003 - 0.034) + \ldots, \\ F_K/F_\pi &= 1 + (0.057 + 0.083) + (-0.005 + 0.045 + 0.021) + \ldots, \\ f_0(\Delta_{K\pi}) &= 1 + (0.055 + 0.083) + (-0.001 + 0.047 + 0.017) + \ldots, \end{aligned} \tag{4.6}$$

and for Fit (IV)

$$\begin{aligned} f_+(0) &= 1 + (-0.027 + 0.008) + (0.012 - 0.002 - 0.029) + \ldots, \\ F_K/F_\pi &= 1 + (0.086 + 0.027) + (-0.005 + 0.078 + 0.009) + \ldots, \\ f_0(\Delta_{K\pi}) &= 1 + (0.083 + 0.026) + (-0.001 + 0.063 + 0.019) + \ldots . \end{aligned} \tag{4.7}$$

The first brackets give the contribution from the loops and the L_i at fourth order and the second brackets represent the one at sixth order from the two-loops, the one-loop with one L_i insertion plus tree graphs with two L_i and the tree graphs $\sim C_i$, in order. One sees that the large contribution of $f_+(0)$ at NNLO is due to big corrections of the dimension six operators, as was the case for the slope and the curvature, see the discussion before. It could be that the corresponding LECs C_i are larger than they are in nature mocking up some higher order effects. The contributions from the two-loop and the one-loop $\sim L_i$ topologies do converge. In the case of F_K/F_π and $f_0(\Delta_{K\pi})$ it is the terms proportional to L_i which are responsible for their not so good convergence in the case of set (b), explaining the difference between the two sets. Let us consider also the convergence of F_K/F_π at larger pion and kaon masses. One has for Fit (IV)

$$\begin{aligned} F_K/F_\pi &= 1 + 0.043 + 0.093 + \ldots = 1.136 + \ldots, \quad \text{set (I)} \\ &= 1 + 0.023 + 0.076 + \ldots = 1.099 + \ldots, \quad \text{set (II)}. \end{aligned} \tag{4.8}$$

For comparison the lattice data are:

$$\begin{aligned} F_K/F_\pi &= 1.134 \pm 0.011, \quad \text{set (I)} \\ &= 1.101 \pm 0.010, \quad \text{set (II)}. \end{aligned} \tag{4.9}$$

As already stated for the scalar form factor and as expected, the convergence gets worse when increasing the values of m_π and m_K. This bad convergence could be an artefact of the use of lattice data obtained at still too high pion and kaon masses for ChPT to really be valid.

5 Conclusion

We have done here a first exploratory study using a two-loop ChPT calculation to fit the lattice data. Certainly finite volume effects for example should be taken into account in a more refined treatment. However, before this can be done, a better knowledge of the L_i are necessary and more lattice data at smaller masses are needed. This is important for checking the convergence of the $SU(3)$ ChPT calculations as well as for a more precise determination of the quantities studied here. Also if the result of set (a) is not very sensitive to the treatment of the NNLO term, see discussion below Eq. (4.1), this is clearly not the case for set (b) and our results here are certainly not the final ones. Indeed, if large values for L_4^r and L_6^r as expected from a large violation of the OZI rule were confirmed in the future then the use of standard ChPT as done here would not really be appropriate. A way of solving the problem could be for example to work within resummed ChPT [58]. A study along this line is in progress [59].

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